Mathematical Logics
First Order Logic*

Fausto Giunchiglia and Mattia Fumagalli

University of Trento

**Originally by Luciano Serafini and Chiara Ghidini*
Modified by Fausto Giunchiglia and Mattia Fumagalli

1 Lecture index

Expressivity of propositional logic - 1

Question

Try to express in Propositional Logic the following statements:

- Mary is a person
- John is a person
- Mary is mortal
- Mary and John are siblings

A solution

Through atomic propositions:

- `Mary-is-a-person`
- `John-is-a-person`
- `Mary-is-mortal`
- `Mary-and-John-are-siblings`

Problem with previous solution

- `Mary-is-a-person`
- `John-is-a-person`
- `Mary-is-mortal`
- `Mary-and-John-are-siblings`

- How do we link `Mary` of the first sentence to `Mary` of the third sentence? Same with `John`.
- How do we link `Mary` and `Mary-and-John`?

Expressivity of propositional logic - II

Question

Try to express in Propositional Logic the following statements:

- All persons are mortal;
- There is a person who is a spy.

A solution

We can give all people a name and express this fact through atomic propositions:

- `Mary-is-mortal` $\wedge$ `John-is-mortal` $\wedge$ `Chris-is-mortal` $\wedge \ldots \wedge$ `Michael-is-mortal`
- `Mary-is-a-spy` $\vee$ `John-is-a-spy` $\vee$ `Chris-is-a-spy` $\vee \ldots \vee$ `Michael-is-a-spy`

The representation is not compact, generalization patterns are difficult to express.

- What is we do not know all the names of the people in our "universe"?
- How can we express the statement independently from the people in the "universe"?

Expressivity of propositional logic - III

Question

Try to express in Propositional Logic the following statements:

- Every natural number is either even or odd

A solution

We can use two families of propositions $even_i$ and odd_i for every $i \geq 1$, and use the set of formulas

$$\{odd_i \vee even_i \mid i \geq 1\}$$

$$\{odd_i \vee even_i \mid i \geq 1\}$$

What happens if we want to state this in one single formula? To do this we would need to write an infinite formula like:

$$(odd_1 \vee even_1) \wedge (odd_2 \vee even_2) \wedge \ldots$$

and this cannot be done in propositional logic.

Expressivity of propositional logic -IV

Question

Express the statements:

- the father of Luca is Italian

Solution (Partial)

- mario-is-father-of-luca $\supset$ mario-is-italian
- michele-is-father-of-luca $\supset$ michele-is-italian
- ...

mario-is-father-of-luca $\supset$ mario-is-italian

michele-is-father-of-luca $\supset$ michele-is-italian

...

This statement strictly depends on a fixed set of people.

What happens if we want to make this statement independently of the set of persons we have in our universe?

Why first order logic?

Because it provides a way of directly representing facts like the following :

1. Mary is a person;
2. John is a person;
3. Mary is mortal;
4. Mary and John are siblings
5. Every person is mortal;
6. There is a person who is a spy;
7. Every natural number is either even or odd;
8. The father of Luca is Italian

... and also to infer the third statement from the first and the fifth.

First order logic (FOL)

Whereas propositional logic assumes world contains facts,
first-order logic (like natural language) assumes the world contains:

- Constants: mary, john, 1, 2, 3, red, blue, world war 1, world war 2, 18th Century. . .
- Predicates: Mortal, Round, Prime, Brother of, Bigger than, Inside, Part of, Has color, Occurred after, Owns, Comes between, . ..
- Functions: Father of, Best friend, Third inning of, One more than, End of, . . .

FOL - Constants and Predicates

- Mary is a person
- John is a person
- Mary is mortal
- Mary and John are siblings

In FOL it is possible to build an atomic propositions by applying a predicate to constants

- *Person*(*mary*)
- *Person*(*john*)
- *Mortal* (*mary*)
- *Siblings*(*mary*, *john*)

FOL - Functions

- The father of Luca is Italian.

In FOL it is possible to build propositions by applying a function to a constant, and then a predicate to the resulting object.

- *Italian*(*fatherOf* (*Luca*))

FOL - Quantifiers and variables

- Every person is mortal;
- There is a person who is a spy;
- Every natural number is either even or odd;

In FOL it is possible to build propositions by applying universal (existential) quantifiers to variables. This allows to quantify to arbitrary objects of the universe.

- $\forall x.Person(x) \supset Mortal(x)$;
- $\exists x.Person(x) \supset Spy(x)$;
- $\forall x.(Odd(x) \vee Even(x))$

Mathematical Logics
First Order Logic*

Fausto Giunchiglia and Mattia Fumagalli

University of Trento

**Originally by Luciano Serafini and Chiara Ghidini*
Modified by Fausto Giunchiglia and Mattia Fumagalli