Mathematical Logic
PL - Reasoning as deduction

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A1	$\varphi \supset (\psi \supset \varphi)$
A2	$(\varphi \supset (\psi \supset \vartheta)) \supset ((\varphi \supset \psi) \supset (\varphi \supset \vartheta))$
A3	$(\neg \psi \supset \neg \varphi) \supset ((\neg \psi \supset \varphi) \supset \varphi)$

Hilbert (propositional) calculus

A1	$\varphi \supset (\psi \supset \varphi)$
A2	$(\varphi \supset (\psi \supset \vartheta)) \supset ((\varphi \supset \psi) \supset (\varphi \supset \vartheta))$
A3	$(\neg\psi \supset \neg\varphi) \supset ((\neg\psi \supset \varphi) \supset \varphi)$
MP	$\dfrac{\varphi \quad \varphi \supset \psi}{\psi}$

$\Gamma \vdash A$ – meaning « A can be deduced/derived from Γ »

- Axioms A1, A2, A3, inference rule MP (for Modus Ponens), assumptions Γ, theorem(s) A , deduction as sequence
- Correctness theorem ($\Rightarrow$) and Completeness theorem ($\Leftarrow$) holds with respect semantics given above, namely

$$\Gamma \vdash \varphi \text{ iff } \Gamma \vDash \varphi$$

Hilbert calculus – example of deduction

Let L be a propositional Language with PROP = {A, B, C}.
Let T = {A, (A → B), (B → C)} theory (set of axioms) which represents intended mental model.
Let C be theorem that we want to prove. Namely we want to prove T |– C .

Proof / deduction
1. A (hypothesis)
2. (A → B) (hypothesis)
3. B (2 applied to 1, via MP)
4. (B → C) (hypothesis)
5. C (4 applied to 3, via MP)

Linear representation of deduction (typical in the literature for Hilbert calculus). As an exercise build tree representation making dependencies explicit

Automatic reasoning based on Hilber Style

- Hilbert style proof system was invented with the main purpose of describing the minimal rational assumptions behind mathematical reasoning.
- Hilber style proofs are supposed to be provided by humans, who can use their intuition to apply smart heuristics to generate them.
- Writing an algorithm that decides on the validity of a formula by searching a Hilbert style proof, is not a good idea.
- We look at alternative ways to write algorithms for deciding the falidity of a FOL formula.

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