

Mathematical Logics

PL - Reasoning via Truth Tables*

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1. Truth tables – summary
2. Decision problems
3. Deciding satisfiability (SAT)
 - a. CNF
 - b. The DPLL SAT decision procedure
4. Satisfiability in practice: MiniSat

Recall of some definitions

- Two formulas F and G are **logically equivalent** (denoted with $F \equiv G$) if for each interpretation I , $I(F) = I(G)$.
- Let F and G be formulas. G is a **logical consequence** of F (denoted with $F \models G$) if each interpretation satisfying F satisfies also G .
- Let F be a formula:
 - F is **valid** if every interpretation satisfies F
 - F is **satisfiable** if F is satisfied by some interpretation
 - F is **unsatisfiable** if there isn't any interpretation satisfying F

Truth Tables: Example

Compute the truth table of $(F \vee G) \wedge \neg (F \wedge G)$.

F	G	$F \vee G$	$F \wedge G$	$\neg (F \wedge G)$	$(F \vee G) \wedge \neg (F \wedge G)$
T	T	T	T	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	F	T	F

Intuitively, what does this formula represent?

Truth Tables: computing validity

Use the truth tables method to determine whether $(p \rightarrow q) \vee (p \rightarrow \neg q)$ is valid.

p	q	$p \rightarrow q$	$\neg q$	$p \rightarrow \neg q$	$(p \rightarrow q) \vee (p \rightarrow \neg q)$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	T	T

The formula is valid since it is satisfied by every interpretation.

Truth Tables: computing satisfiability

Use the truth tables method to determine whether $(\neg p \vee q) \wedge (q \rightarrow \neg r \wedge \neg p) \wedge (p \vee r)$ (denoted with F) is satisfiable.

p	q	r	$\neg p \vee q$	$\neg r \wedge \neg p$	$q \rightarrow \neg r \wedge \neg p$	$(p \vee r)$	F
T	T	T	T	F	F	T	F
T	T	F	T	F	F	T	F
T	F	T	F	F	T	T	F
T	F	F	F	F	T	T	F
F	T	T	T	F	F	T	F
F	T	F	T	T	T	F	F
F	F	T	T	F	T	T	T
F	F	F	T	T	T	F	F

There exists an interpretation satisfying F , thus F is satisfiable.

Truth Tables: computing logical consequence

Use the truth tables method to determine whether $p \wedge \neg q \rightarrow p \wedge q$ is a logical consequence of $\neg p$.

p	q	$\neg p$	$p \wedge \neg q$	$p \wedge q$	$p \wedge \neg q \rightarrow p \wedge q$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	F	T
F	F	T	F	F	T

Truth Tables: computing logical equivalence

Use the truth tables method to determine whether $p \rightarrow (q \wedge \neg q)$ and $\neg p$ are logically equivalent.

p	q	$q \wedge \neg q$	$p \rightarrow (q \wedge \neg q)$	$\neg p$
T	T	F	F	F
T	F	F	F	F
F	T	F	T	T
F	F	F	T	T

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