

Mathematical Logics

Set Theory*

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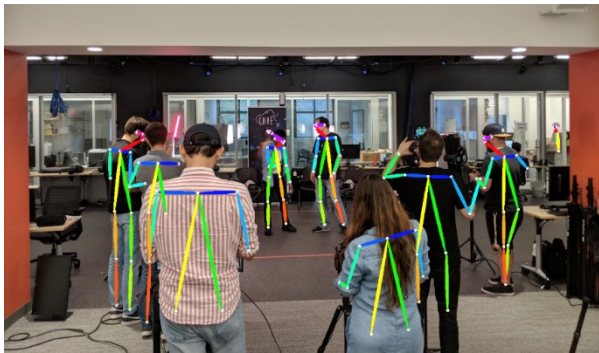


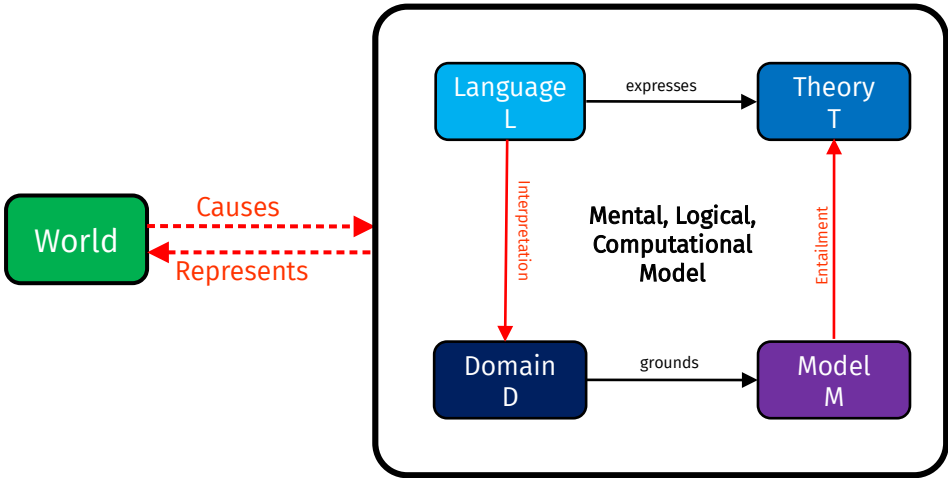
**Originally by Luciano Serafini and Chiara Ghidini
Modified by Fausto Giunchiglia and Mattia Fumagalli
Further changes by Adolfo Villafiorita*

1. Introduction and motivation
2. Basic notions
3. Relations
4. Functions
5. Exercises

Exercise 1 (starting from perception)

Suppose we have an Artificial Intelligence (AI) system whose task is to extract, by means of Computer Vision Algorithms, semantic information from images (with the goal of enabling search and reasoning services over visual information)



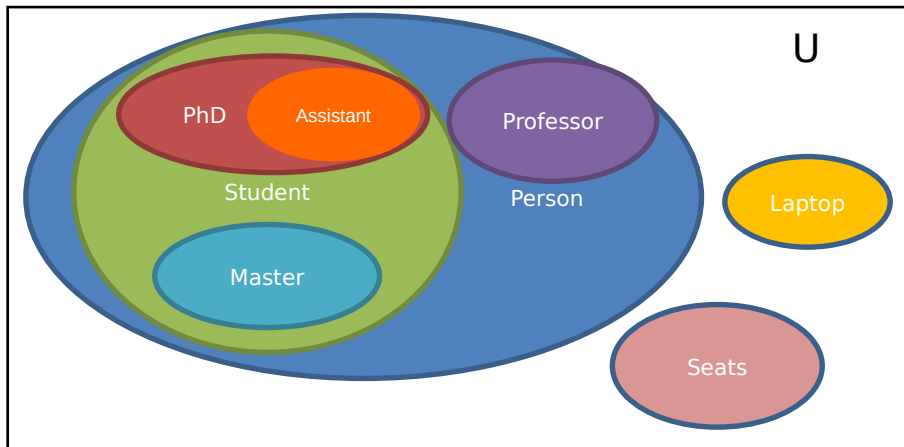


Conceptual modelling phase

- In the class we have students (master? PHD?), a professor, seats, laptops. There may be a blackboard... and many other items...
- All the classes group a set of objects (how many objects?)
- All these objects are linked with some kinds of relations (denoting a certain kind of event): what are the properties of these relations?

Exercise 1 (starting from perception)

Committing to set theory, an example of Venn Diagram



Exercise 1 (starting from perception)

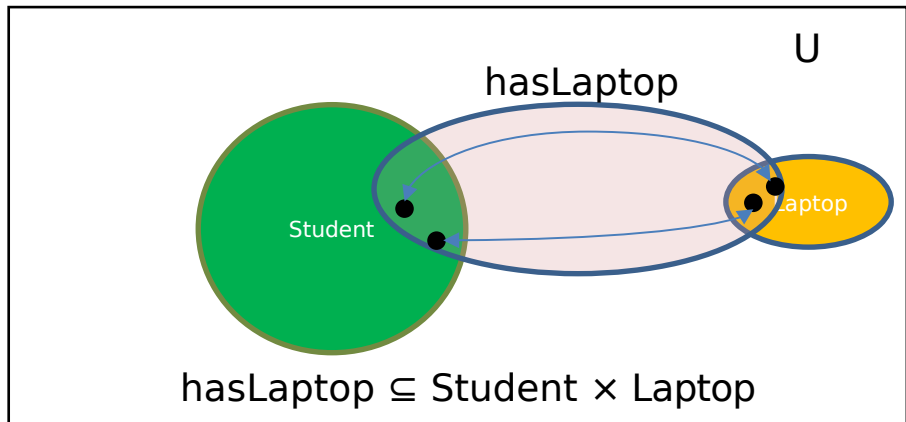
Defining the language

- Constants: Fausto, Mary, Paul, Jane, Mac1, Win1, Thinkpad1
- Sets: Person, Professor, Student, Laptop, Seat, Macs, ThinkPads
- Binary relations: has-laptop, has-seat, to-the-right
- N-ary relations: teaches, in-between

Domain = {1, 2, 3, 4, ... }

Exercise 1 (starting from perception)

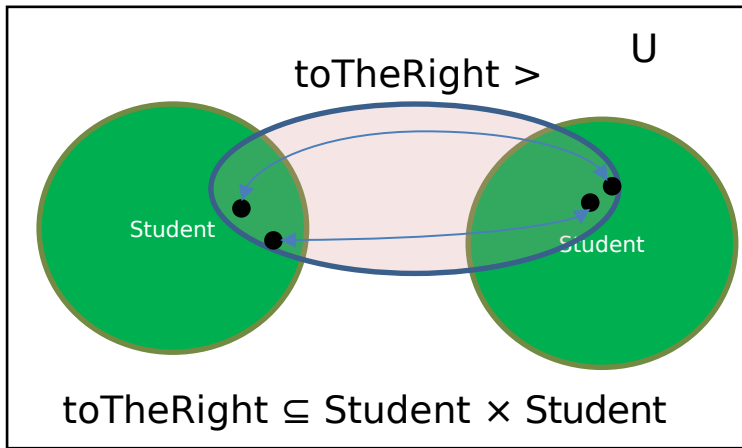
Committing to set theory, modeling relations (example 1)



$I(\mathbf{has-laptop}) = \{(Mary, ThinkPad\ 1), \dots\}$

Exercise 1 (starting from perception)

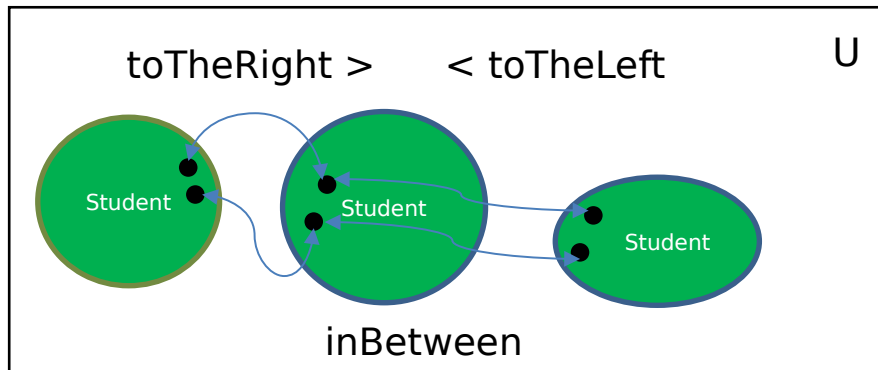
Committing to set theory, modelling relations (example 2)



$$I(\text{to-the-right}) = \{(\text{Mary}, \text{Paul}), \dots\}$$

Exercise 1 (starting from perception)

Committing to set theory, modeling relations (example 2, extended)



$\text{InBetween} \subseteq \text{Student} \times \text{Student} \times \text{Student}$

$I(\text{to-the-right}) = \{(\text{Mary}, \text{Paul}), \dots\}$

$I(\text{to-the-left}) = \{(\text{Jane}, \text{Paul}), \dots\}$

$I(\text{in-between}) = \{(\text{Mary}, \text{Paul}, \text{Jane}), \dots\}$

Exercise 1 (starting from perception)

Committing to set theory, modeling relations (example 2, extended)

- **in-between**(Mary, Paul, Jane)
- **to-the-right**(Mary, Paul)
- **to-the-left**(Jane, Paul)

where **to-the-right**(x,y) can be defined as **to-the-left**(y,x) and **to-the-right/to-the-left** are **transitive relations, such that** (for instance):

toTheRight(Mary, Paul) and **toTheRight**(Paul, Jane)
then **toTheRight**(Paul, Jane)

Exercise 1 (starting from perception)

A possible Model

$I(\textit{fausto}) = 1$, $I(\textit{mary}) = 2$, $I(\textit{paul}) = 3$,
 $I(\textit{jane}) = 4$, $I(\textit{mac1}) = 5$, $I(\textit{win1}) = 6$,
 $I(\textit{logic}) = 7$, $I(\textit{math}) = 8$, $I(\textit{kdi}) = 9$

$I(\textit{has-Laptop}) = \textit{hL}$ $\textit{hL}(2) = 5$

$I(\textit{teaches}) = \textit{T}$

$\textit{hL}(4) = 6$

$\{\langle 1, 7, 9 \rangle, \langle 1, 9, 8 \rangle\}$

Modelling a meeting

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"At the beginning of the meeting you were all like
"What's with the toaster?! Where's your laptop?!"
But now..."