Mathematical Logics
Propositional Logic *

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Reference(s):

- **Francesco Berto, Logica da zero a Gödel, Laterza, 2018 (capitolo 1)**

**Originally by Luciano Serafini and Chiara Ghidini*
Modified by Fausto Giunchiglia and Mattia Fumagalli

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Propositional logic language

Definition (Propositional alphabet)

Logical symbols $\neg, \wedge, \vee, \supset, \equiv$

Non logical symbols A set $\mathbf{PROP}$ of symbols P called propositional variables

Separator symbols "(" and ")"

Definition (Well formed formulas (or simply formulas))

- every $P \in \mathbf{PROP}$ is an atomic formula
- every atomic formula is a formula
- if A and B are formulas then $\neg A$, $A \wedge B$, $A \vee B$ $A \supset B$, and $A \equiv B$ are formulas

Propositional variables and constants

- Propositional constants:
  - "B. Obama is drinking a bier"
  - "The U.S.A. president is drinking a bier",
  - "B. Obama si sta facendo una birra"

- Propositional variables: A, B, C, AA, …, where a propositional variable can be substitued with a propositional constant, a propositional variable or in general any well formed (propositional) formula

Formulas cont'd

Example ((non) formulas)

Formulas	Non formulas
$P \longrightarrow Q$	$P\ Q$
$P \longrightarrow (Q \longrightarrow R)$	$(P \longrightarrow \wedge ((Q \longrightarrow R)$
$P \wedge Q \longrightarrow R$	$P \wedge Q \longrightarrow \neg R \neg$

Reading formulas

Problem

How do we read the formula $P \wedge Q \rightarrow R$?

The formula $P \wedge Q \rightarrow R$ can be read in two ways:

1. $(P \wedge Q) \rightarrow R$
2. $P \wedge (Q \rightarrow R)$

Symbol priority

$\neg$ has higher priority, then $\wedge, \vee, \rightarrow, \equiv$ with decreasing priority

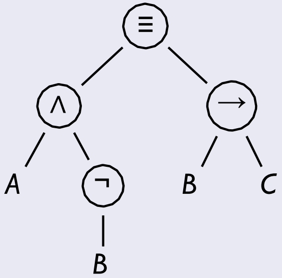
Parenthesis can be used to stress or change the priority.

Symbol	Priority
$\neg$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\equiv$	5

Formulas as trees

A formula can be seen as a tree. Leaf nodes are associated to propositional variables, while intermediate (non-leaf) nodes are associated to connectives.

For instance the formula $(A \wedge \neg B) \equiv (B \rightarrow C)$ can be represented as the tree



Subformulas

Definition

(Proper) Subformula

- A is a subformula of itself
- A and B are subformulas of $A \wedge B$, $A \vee B$ $A \supset B$, e$A \equiv B$
- A is a subformula of $\neg A$
- if A is a subformula of B and B is a subformula of C, then A is a subformula of C.
- A is a proper subformula of B if A is a subformula of B and A is different from B.

Remark

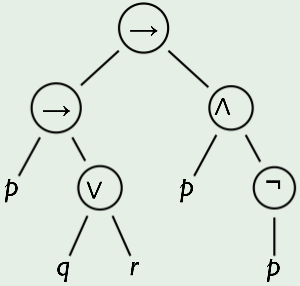
The subformulas of a formula represented as a tree correspond to all the different subtrees of the tree associated to the formula, one for each node.

Subformulas

Example

The subformulas of $(p \to (q \vee r)) \to (p \wedge \neg p)$ are

$(p \to (q \vee r)) \to (p \wedge \neg p)$
$(p \to (q \vee r))$
$p \wedge \neg p$
p
$\neg p$
$q \vee r$
q
r



Proposition

Every formula has a finite number of subformulas

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