Mathematical Logics
Set Theory*

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1. Introduction and motivation

2. Basic notions

3. Relations

4. Functions

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Functions

Given two sets A and B, a function f from A to B is a relation that associates to each element a in A exactly one element b in B. Denoted with
$f : A \rightarrow B$

The domain of f is the whole set A; the image of each element a in A is the element b in B s.t. $b = f(a)$; the co-domain of f (or image of f) is a subset of B defined as follows:
$Im_f = \{b \in B \mid \text{there exists an } a \in A \text{ s.t. } b = f(a)\}$

Notice that: it can be the case that the same element in B is the image of several elements in A.

Classes of Functions

A function $f{:}A \to B$ is surjective if each element in B is image of some elements in A: for each $b \in B$ there exists an $a \in A$ s.t. $f(a) = b$

A function $f{:}A \to B$ is injective if distinct elements in A have distinct images in B:
for each $b \in Im_f$ there exists a unique $a \in A$ s.t. $f(a) = b$

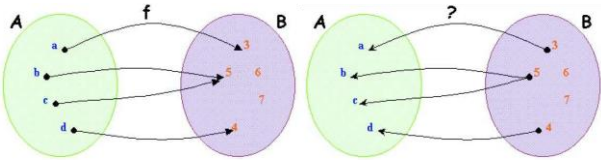
A function $f{:}A \to B$ is bijective if it is injective and surjective:
for each $b \in B$ there exists a unique $a \in A$ s.t. $f(a) = b$

If $f : A \to B$ is bijective we can define its inverse function:

$$f^{-1} : B \to A$$

For each function f we can define its inverse relation; such a relation is a function iff f is bijective.

Example:



the inverse relation of f is NOT a function.

Composed Function

Let $f : A \to B$ and $g : B \to C$ be functions. The composition of f and g is the function $g \circ f : A \to C$ obtained by applying f and then g:

$$(g \circ f)(a) = g(f(a)) \text{ for each } a \in A$$
$$g \circ f = \{(a, g(f(a)) \mid a \in A)\}$$

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