

# Mathematical Logics

## Set Theory\*

Fausto Giunchiglia and Mattia Fumagalli

University of Trento



*\*Originally by Luciano Serafini and Chiara Ghidini  
Modified by Fausto Giunchiglia and Mattia Fumagalli*

## 1. Introduction and motivation

2. Basic notions

3. Relations

4. Functions

5. Exercises

## Extensional Semantics: Extensions

- ❑ The meanings which are intended to be attached to the symbols and propositions form the **intended interpretation** of the language
- ❑ We consider only **extensional (semantics)**
- ❑ The extensional semantics of  $L$  is based on the notion of “extension” of a formula (proposition) in  $L$
- ❑ The **extension of a proposition** is the **totality**, or **class**, or **set** of all objects  $D$  (domain elements) to which the proposition applies

# Domain in set theory – example I

## Language L

- Alphabet =  $\{A, B, \wedge, \vee\}$

- Syntactic constructors:

  - If A, B formulas in L then  $A \wedge B$  is also a formula in L

  - If A, B formulas of L then  $A \vee B$  is also a formula in L

- Domain D

  - $D = \{T, F\}$

- Interpretation I:  $I: L \rightarrow D$

# Domain in set theory – example 2

## Language L

- ❑ **Alphabet** =  $\{A, B, \wedge, \vee\}$
- ❑ **Syntactic constructors:**
  - ❑ If A, B formulas in L then  $A \wedge B$  is also a formula in L
  - ❑ If A, B formulas of L then  $A \vee B$  is also a formula in L
- ❑ **Domain D** – In our course always set theory
  - ❑  $D = \{\text{Fausto, Maria, John, ...}, 1, 2, 10, \dots\}$
- ❑ **Interpretation I:**  $I: L \rightarrow D$

# Mathematical Logics

## Set Theory\*

Fausto Giunchiglia and Mattia Fumagalli

University of Trento



*\*Originally by Luciano Serafini and Chiara Ghidini  
Modified by Fausto Giunchiglia and Mattia Fumagalli*