Mathematical Logics
Description Logic: Tableaux

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DL as Query Language - intuition

TBox = {}
ABox = {Child(John, Mary), Female(Mary)}

NL Query: Who are the individuals having only female children?

DL Query: T, A ⊨ ∀Child.Female

Answer: {John}

❑<u>ABox services</u> are generally applied to resolve a query

How to use ABox Reasoning Services

ABox Service	Description	Query
Instance retrieval	Given a concept C, retrieve all the instances a which satisfy C w.r.t. the ABox A.	$A \vDash C$
Instance checking	Check whether an assertion C(a) is entailed by the ABox, i.e. check whether a belongs to C.	$A \vDash C(a)$ $A \vDash R(a,b)$

NOTE: this means that before answering we need to expand the ABox (w.r.t. the TBox) and reason on the identified model (see before)

TBox = {Horse $\sqsubseteq$ Animal, Mule $\sqsubseteq$ Animal}

ABox = {Horse(Furia), Parent(Speedy, Furia)}

NL Query: Is Furia an animal?

DL Query: T, A $\models$ Animal(Furia)

YES, in fact the ABox can be expanded as follows:

ABox = {Horse(Furia), Animal(Furia), Parent(Speedy, Furia)}

TBox = {Horse $\sqsubseteq$ Animal $\sqcap$ $\neg$Mule, Mule $\sqsubseteq$ Animal}
ABox = {Horse(Furia), Parent(Speedy, Furia)}

NL Query: Is Furia a mule?

DL Query: T, A $\vDash$ Mule(Furia)

NO, in fact the ABox can be expanded as follows:
ABox = {Horse(Furia), Animal(Furia), $\neg$Mule(Furia),
Parent(Speedy, Furia)}

TBox = {Horse ⊑ Animal, Mule ⊑ Animal}
ABox = {Horse(Furia), Parent(Speedy, Furia)}

NL Query: Is Furia a mule?

DL Query: T, A ⊨ Mule(Furia)

NO (BY CLOSED WORLD ASSUMPTION), in fact the ABox can be expanded as follows:

ABox = {Horse(Furia), Animal(Furia), Parent(Speedy, Furia)}

If we drop closed world assumption the answer should be: I DO NOT KNOW

TBox = {Horse ⊑ Animal, Mule ⊑ Animal}
ABox = {Horse(Speedy), Horse(Furia), Parent(Speedy, Furia)}

NL Query: Is there any animal which is not a horse nor a mule, and is parent of a horse?

DL Query: T, A ⊨ ∃Parent.Horse ⊓ ¬ (Horse ⊓ Mule)

i.e. is the formula satifiable?

TBox = {Horse ⊑ Animal, Mule ⊑ Animal}
ABox = {Horse(Speedy), Horse(Furia), Parent(Speedy, Furia)}

Is ∃Parent.Horse ⊓ ¬ (Horse ⊓ Mule) satifiable?

⊓-rule A' = { ∃Parent.Horse(x), ¬(Horse ⊓ Mule)(x)}

∃-rule A' = {Horse(Furia), Parent(Speedy, Furia), (¬Horse ⊔ ¬Mule)(x)}

⊔-rule A' = {Horse(Furia), Parent(Speedy, Furia), ¬Horse(Furia)}

<div align="right">inconsistent!</div>

      or

      A' = {Horse(Furia), Parent(Speedy, Furia), ¬Mule(Furia)}

<div align="right">consistent!</div>

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