Mathematical Logics
Description Logic: Tableaux

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Lecture index

1. Idea: DL is a MultiModal Modal Logic
2. DL reasoning as MultiModal SAT reasoning
3. Examples: TBOX reasoning
4. Examples: ABOX reasoning – DL as a query language

The Tableau Algorithm (same as Modal Logics)

- The formula C in input is translated into Negation Normal Form (negation pushed down to negate atomic formulas, useful to avoid negation rules).

- An ABox A is incrementally constructed by adding assertions according to the constraints in C (identified by decomposition) following precise transformation rules

- Each time we have more than one option we split the space of the solutions as in a decision tree (i.e. in presence of ⊔)

- When a contradiction is found (i.e. A is inconsistent) we need to try another path in the space of the solutions (backtracking)

- The algorithm stops when either we find a consistent A satisfying all the constraints in C (the formula is satisfiable) or there is no consistent A (the formula is unsatisfiable)

Transformation rules

❑ ⊓-rule

Condition: A contains $(C1 \sqcap C2)(x)$, but not both $C1(x)$ and $C2(x)$

Action: $A' = A \cup \{C1(x), C2(x)\}$

$T=\{Mother \equiv Female \sqcap \exists hasChild.Person\}$ $A=\{Mother(Anna)\}$
Is $(\neg\exists hasChild.Person \sqcap \neg\exists hasParent.Person)$ satisfiable?

Expand A w.r.t. T
$Mother(Anna) \Rightarrow (Female \sqcap \exists hasChild.Person)(Anna) \Rightarrow$
$A' = A \cup \{Female(Anna), (\exists hasChild.Person)(Anna)\}$

$(\neg\exists hasChild.Person \sqcap \neg\exists hasParent.Person)(Anna) \Rightarrow$
$A' = A \cup \{(\neg\exists hasChild.Person)(Anna), (\neg\exists hasParent.Person)(Anna)\}$
Both of them must be true, but the first constraint is clearly in contradiction with A'

NOTE: See use of constants/ variables as arguments

Transformation rules

❑ ⊔-rule

Condition: A contains $(C1 \sqcup C2)(x)$, but neither $C1(x)$ or $C2(x)$

Action: $A' = A \cup \{C1(x)\}$ and $A'' = A \cup \{C2(x)\}$

T={Parent≡∃hasChild.Female⊔∃hasChild.Male,
Person≡Male⊔Female, Mother≡Parent ⊓Female}
A={Mother(Anna)}
Is ¬∃hasChild.Person satisfiable?

Expand A w.r.t. T
A = {Mother(Anna)} ⇨ A’ = A ∪ {Parent(Anna), Female(Anna)}
Parent(Anna) ⇨ (∃hasChild.Female⊔∃hasChild.Male)(Anna) ⇨
(∃hasChild.Female)(Anna) or (∃hasChild.Male)(Anna)

Both are in contradiction with ¬∃hasChild.Person, not satisfiable.

Transformation rules

❑ ∃-rule

Condition: A contains $(\exists R.C)(x)$, but there is no z such that both $C(z)$ and $R(x,z)$ are in A

Action: $A' = A \cup \{C(z), R(x,z)\}$

$T=\{Parent \equiv \exists hasChild.Female \sqcup \exists hasChild.Male,$
$Person \equiv Male \sqcup Female, Mother \equiv Parent \sqcap Female\}$
$A=\{Mother(Anna), hasChild(Anna,Bob), \neg Female(Bob)\}$
Is $\neg(\exists hasChild.Person)$ satisfiable?

Expand A w.r.t. T
Mother(Anna) ⇨ Parent(Anna) ⇨
$(\exists hasChild.Female \sqcup \exists hasChild.Male)(Anna)$

take $(\exists hasChild.Male)(Anna)$ ⇨ hasChild(Anna,Bob), Male(Bob) …

Transformation rules

❑∀-rule

Condition: A contains (∀R.C)(x) and R(x,z), but it does not contain C(z)

Action: A' = A ∪ {C(z)}

T={DaughterParent≡∀hasChild.Female, Male⊓Female⊑⊥}
A={hasChild(Anna,Bob), ¬Female(Bob)}
Is DaughterParent satisfiable?

Expand A w.r.t. T
DaughterParent(x) ⇨ ∀hasChild.Female(x) ⇨

Given that hasChild(Anna,Bob) ⇨ A' = A ∪ {Female(Bob)}

but this in contradiction with ¬Female(Bob)

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