Mathematical Logics
Description Logic: Tbox and Abox

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Terminological axioms

- Two new logical symbols

  $\sqsubseteq$ (subsumption), with $I \models C \sqsubseteq D$ iff $I(C) \subseteq I(D)$

  $\equiv$ (equivalence), with $I \models C \equiv D$ iff $I(C) = I(D)$

Inclusion axioms

$C \sqsubseteq D$ has to be read "C is subsumed by (more specific than / less general than) D"

Equality axioms

$C \equiv D$ has to be read "C is equivalent to D"

NOTE: $C \equiv D$ holds iff both $C \sqsubseteq D$ and $D \sqsubseteq C$ hold

NOTE:

- if $C \sqsubseteq D$, we can use the symbol $\sqsupseteq$ to say that $D \sqsupseteq C$
- $D \sqsupseteq C$ to be read as "D subsumes (less specific than / more general than) C"

Examples of terminological axioms

- **Inclusion Axioms (inclusions)**

  $\text{Master} \sqsubseteq \text{Student}$
  $\text{Woman} \sqsubseteq \text{Person}$
  $\text{Woman} \sqcup \text{Father} \sqsubseteq \text{Person}$

- **Equality Axioms (equalities)**

  $\text{Student} \equiv \text{Pupil}$
  $\text{Parent} \equiv \text{Mother} \sqcup \text{Father}$
  $\text{Woman} \equiv \text{Person} \sqcap \text{Female}$

Tbox - Definitions and specializations

- A definition is an equality with an atomic concept on the left hand

Bachelor	$\equiv$ Student $\sqcap$ Undergraduate
Woman	$\equiv$ Person $\sqcap$ Female
Parent	$\equiv$ Mother $\sqcup$ Father

- A specialization is an inclusion with an atomic concept on the left hand

Student	$\sqsubseteq$ Person $\sqcap$ Study
PhD	$\sqsubseteq$ Student $\sqcap$ Lecturer

Tbox - Terminology

- A terminology (or TBox) is a set of definitions and specializations

Woman	≡ Person ⊓ Female
Man	≡ Person ⊓ ¬Woman
Student	⊑ Person ⊓ Study
Bachelor	≡ Student ⊓ Undergraduate
PhD	⊑ Student ⊓ Lecturer

- Terminological axioms express constraints on the concepts of the language, i.e. they limit the possible models

- The TBox is the set of all the constraints on the possible models

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