Mathematical Logics
Description Logic: Tbox and Abox

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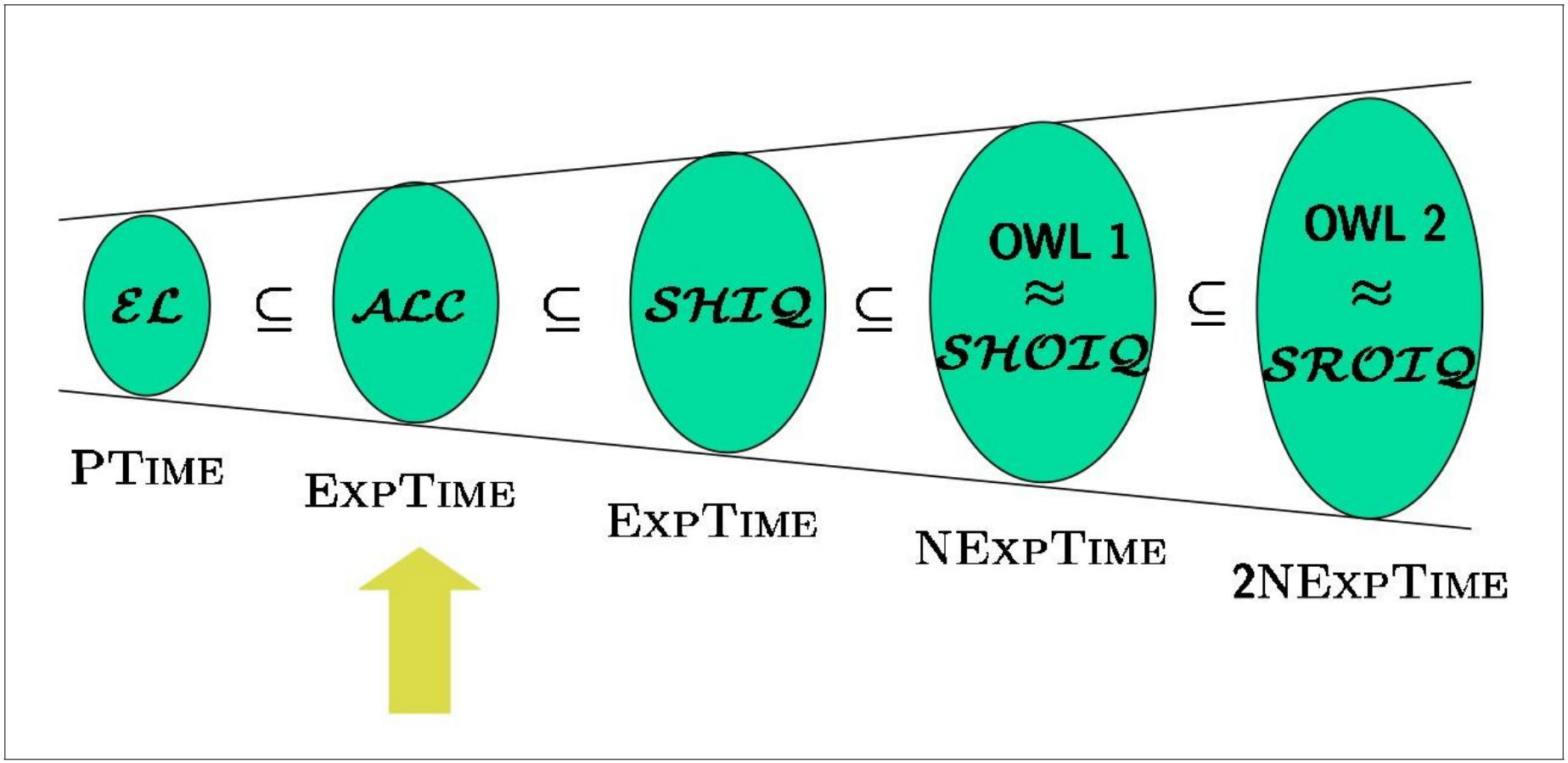
Lecture index

1. Families of Description Logics
2. TBOX: syntax and semantics
3. TBOX: terminology
4. TBOX: reasoning
5. ABOX: syntax and semantics
6. ABOX: reasoning
7. Closed World Assumption (CWA) and Open World Assumption (OWA)

Many description logics



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Syntax – ALC (AL with full concept negation)

Formation rules:

<Atomic> ::= A | B | ... | P | Q | ... | ⊥ | ⊤

<wff> ::= <Atomic> | ¬ <wff> | <wff> ⊓ <wff> | <wff> ⊔ <wff> | ∀R.C | ∃R.C

Examples

- ¬ (Mother ⊓ Father): "it cannot be both a mother and father"
- Person ⊓ Female: "persons that are female"
- Person ⊓ ∃hasChild. ⊤: "(all those) persons that have a child"
- Person ⊓ ∀hasChild. ⊥: "(all those) persons without a child"
- Person ⊓ ∀hasChild.Female: "persons all of whose children are female"

Syntax - *AL** Interpretation (Δ,I)

- $I(\bot) = \emptyset$ and $I(\top) = \Delta$ (full domain, "Universe")
- For every concept name A of L, $I(A) \subseteq \Delta$
- $I(\neg C) = \Delta \setminus I(C)$
- $I(C \sqcap D) = I(C) \cap I(D)$
- $I(C \sqcup D) = I(C) \cup I(D)$

- For every role name R of L, $I(R) \subseteq \Delta \times \Delta$
- $I(\forall R.C) = \{a \in \Delta \mid \text{for all } b, \text{ if } (a,b) \in I(R) \text{ then } b \in I(C)\}$
- $I(\exists R.\top) = \{a \in \Delta \mid \text{exists } b \text{ s.t. } (a,b) \in I(R)\}$
- $I(\exists R.C) = \{a \in \Delta \mid \text{exists } b \text{ s.t. } (a,b) \in I(R), b \in I(C)\}$
- $I(\geq nR) = \{a \in \Delta \mid |\{b \mid (a, b) \in I(R)\}| \geq n\}$
- $I(\leq nR) = \{a \in \Delta \mid |\{b \mid (a, b) \in I(R)\}| \leq n\}$

NOTE: last two elements not in ALC

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