

CL-2020 - ML Informal to Formal

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Recap

Modal Logics introduce two modalities, which predicate about the truth of formulas in reachable worlds:

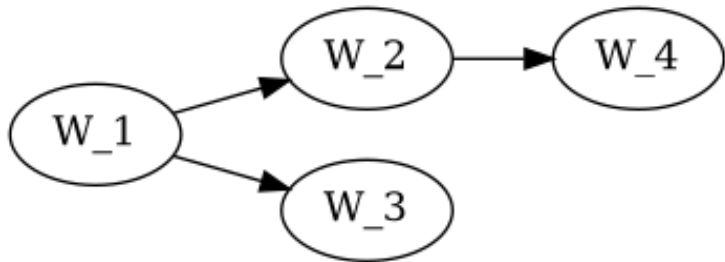
- ▶ **possibility** $\diamond P$, P is true in one of the reachable worlds
- ▶ **necessity** $\square P$, P is true in all reachable worlds

“Reachable worlds” are defined as follows:

- ▶ $F = \langle W, R \rangle$ (W is a set and $R : W \rightarrow W$)
- ▶ $M = \langle F, I \rangle$ ($I : P \rightarrow 2^W$)

So true

Truth is defined relative to a world, validity relative to worlds and frames.



- ▶ If $I(P) = \{W_2, W_3\}$, then $\Box P$ is true in W_1
- ▶ If $I(Q) = \{W_2\}$, then $\Diamond Q$ is true in W_1

Flavours

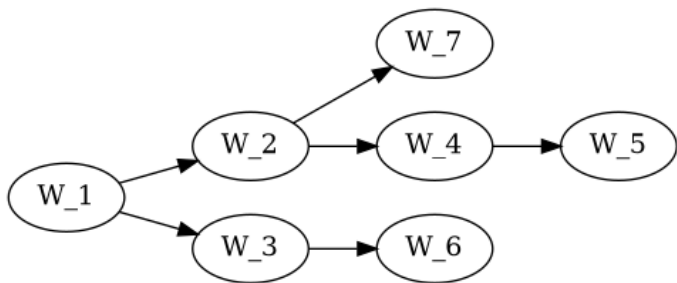
Modal logics come in different flavours to express different kind of possibilities and necessity:

- ▶ **Alethic** (\Box, \Diamond) and **Deontic** (O, P, F) logics about what is necessary, possible, “right” or “wrong”
- ▶ **Temporal** (G, F), **Dynamic**, and **Computational** (AX, AG, AF, A.U., EX) logics predicate about evolution over time
- ▶ **Epistemic** (B_a, K_a) and **contextual** logics about knowledge from a point of view

Informal to formal: simple Propositions

Simple formulas

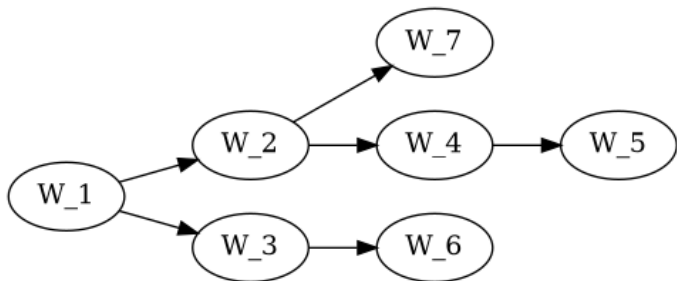
- ▶ “Piove” ... “ P ”
- ▶ “Potrebbe piovere” ... “ $\diamond P$ ”
- ▶ “Pioverà (per certo)” ... “ $\square P$ ”
- ▶ “Il bianco vince in tre mosse”: $\square\square\square W$
- ▶ “Il bianco potrebbe vincere in tre mosse”: $\diamond\diamond\diamond W$
- ▶ “Non c'è futuro”: $\square \perp$



Informal to formal: Implications

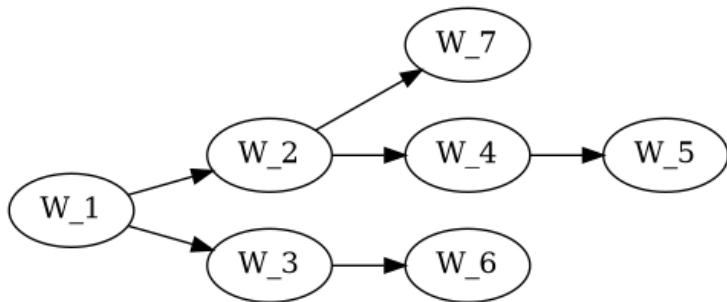
Implications:

- ▶ “Se non piove oggi, potrebbe piovere domani”: $\neg P \supset \diamond P$
- ▶ “Se non piove oggi, pioverà domani”: $\neg P \supset \Box P$
- ▶ “Se studi, avrai successo”: $Study \supset \Box Success$
($Study \supset \diamond Success?$)
- ▶ “Se il nero fa un errore alla prossima mossa, il bianco vince”:
 $\diamond \Box W, \diamond (B_e \supset \Box W)$



Informal to formal: Other formulas

- ▶ “Il sole sorge”: $S \wedge \Box S \wedge \Box \Box S \wedge \dots$
- ▶ “È vietato uscire”: $OHome$
- ▶ “È vietato uscire di sera”: $E \supset OH$



Informal to formal: Beliefs

- ▶ “Credo che stia piovendo”: $B_{me}P$
- ▶ “Sono sicuro che stia piovendo”: $K_{me}P$
- ▶ “Credo che tu sappia che stia piovendo”: $B_{me}K_{you}P$
- ▶ “So che pensi che gli alieni esistano, ma non è così”:
 $K_{me}B_{you}A \wedge \neg A$

