Mathematical Logics
Modal Logic: K and more*

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Multi-Modal Logics

All the definitions given for basic modal logic can be generalized in the case in which we have n $\Box$-operators $\Box_1, \ldots, \Box_n$ (and also $\Diamond_1, \ldots, \Diamond_n$), which are interpreted in the frame

$$F = (W, R_1, \ldots R_n)$$

Every $\Box_i$ and $\Diamond_i$ is interpreted w.r.t. the relation R_i .

A logic with n modal operators is called Multi-Modal. Multi-Modal logics are often used to model **Multi-Agent systems** where modality $\Box_i$ is used to express the fact that "agent i knows (believes) . . . ".

Exercises

Exercise

Let $F = (W, R_1, \ldots, R_n)$ be a frame for the modal language with n modal operator $\Box_1, \ldots, \Box_n$. Show that the following properties holds:

1. $F \models \mathbf{K}_i$ (where $\mathbf{K}_i$ is obtained by replacing $\Box$ with $\Box_i$ in the axiom $\mathbf{K}$)
2. If $R_i \subseteq R_j$ then $F \models \Diamond_i\varphi \supset \Diamond_j\varphi$
3. If $R_i \subseteq R_j$ then $F \models \Box_j\varphi \supset \Box_i\varphi$
4. If $R_i \subseteq R_j \circ R_k$, then[a] $F \models \Diamond_i\varphi \supset \Diamond_j\Diamond_k\varphi$

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[a] Given two binary relations R and S on the set W,

$$R \circ S = \{(v, u) | (v, w) \in R \text{ and } (w, u) \in S\}$$

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