

# Mathematical Logics

## Modal Logic: K and more\*

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Modified by Fausto Giunchiglia and Mattia Fumagalli*

1. Calculi for modal logics
2. Modal K (Hilbert calculus)
3. Properties of accessibility relation and modal axioms
4. Modal KT
5. Modal KB
6. Modal KD
7. Modal KT4 = S4
8. Modal KT5 = S5
9. MultiModal Logics
10. Multiagent Knowledge and belief

## The axiom **D**

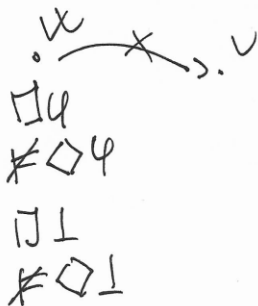
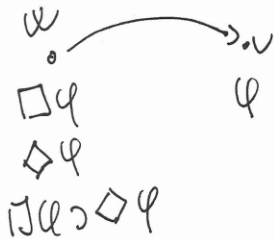
If a frame is serial then the formula

$$\mathbf{D} \quad \Box\varphi \supset \Diamond\varphi$$

holds.

IF A FRAME  $F$  IS SERIAL THEN  $F \models \exists \varphi \supset \forall \varphi$

~~$\exists w. \exists v R(w, v)$~~



# R is serial - soundness

Let  $M$  be a model on a serial frame  $F = (W, R)$  and  $w$  any world in  $W$ .  
We prove that  $M, w \models \Box\varphi \supset \Diamond\varphi$ .

- 1 Since  $R$  is serial there is a world  $w' \in W$  with  $wRw'$
- 2 Suppose that  $M, w \models \Box\varphi$  (Hypothesis)
- 3 From the satisfiability condition of  $\Box$ ,  $M, w \models \Box\varphi$  implies that  $M, w' \models \varphi$
- 4 Since there is a world  $w'$  accessible from  $w$  that satisfies  $\varphi$ , from the satisfiability conditions of  $\Diamond$  we have that  $M, w \models \Diamond\varphi$  (Thesis) .
- 5 Since from (Hypothesis) we have derived (Thesis), we can conclude that  
 $M, w \models \Box\varphi \supset \Diamond\varphi$ .

# R is serial - completeness

(By absurde). Suppose that a frame  $F = (W, R)$  is not Serial.

- 1 If  $R$  is not serial then there is a  $w \in W$  which does not have any accessible world. I.e., for all  $w'$  it does not hold that  $wRw'$ .
- 2 Let  $M$  be any model on  $F$ .
- 3 Form the satisfiability condition of  $\Box$  and from the fact that  $w$  does not have any accessible world, we have that  $M, w \models \Box\varphi$ .
- 4 Form the satisfiability condition of  $\Diamond$  and from the fact that  $w$  does not have any accessible world, we have that  $M, w \models \neg\Diamond\varphi$ .
- 5 this contradicts the hypothesis and therefore  $M, w \models \Box\varphi \supset \Diamond\varphi$

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