Mathematical Logics
Modal Logic: K and more*

Fausto Giunchiglia and Mattia Fumagallli

University of Trento

*Originally by Luciano Serafini and Chiara Ghidini
Modified by Fausto Giunchiglia and Mattia Fumagalli

Lecture index

1. Calculi for modal logics
2. Modal K (Hilbert calculus)
3. Properties of accessibility relation and modal axioms
4. Modal KT
5. Modal KB
6. Modal KD
7. Modal KT4 = S4
8. Modal KT5 = S5
9. MultiModal Logics
10. Multiagent Knowledge and belief

R is symmetric

The axiom **B**

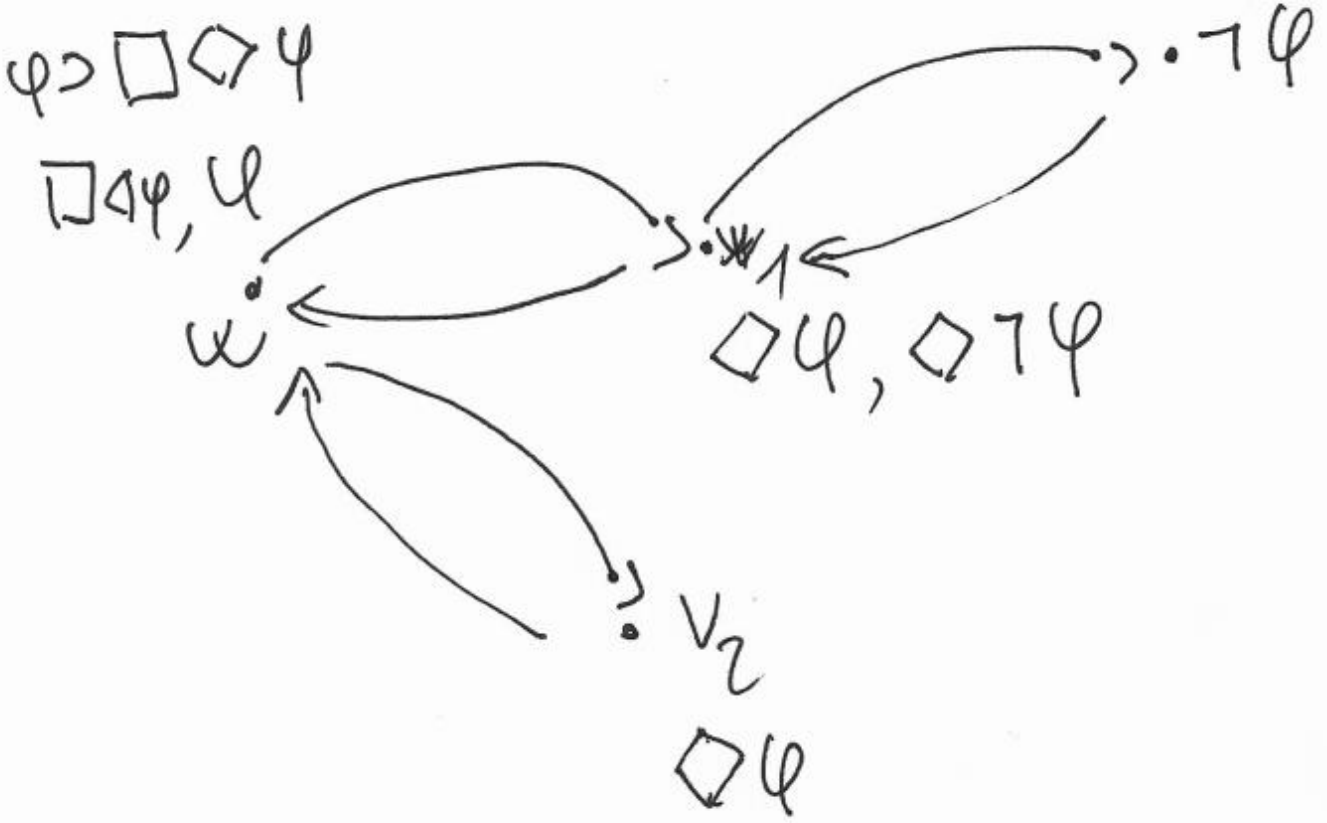
If a frame is symmetric then the formula

$$\mathbf{B} \quad \varphi \supset \Box\Diamond\varphi$$

holds.

A FRAME $\mathcal{F}$ is symmetric $\Rightarrow \mathcal{F} \models \varphi \supset \Box\Diamond\varphi$

$$\forall w\, v.\, (R(w,v) \supset R(v,w))$$



R is symmetric - soundness

Let M be a model on a symmetric frame $F = (W, R)$ and w any world in W. We prove that $M, w \vDash \varphi \supset \Box\Diamond\varphi$.

1. Suppose that $M, w \vDash \varphi$ (Hypothesis)
2. we want to show that $M, w \vDash \Box\Diamond\varphi$ (Thesis)
3. Form the satisfiability conditions of $\Box$, we need to prove that for every world w'

   accessible from w, $M, w' \vDash \Diamond\varphi$.
4. Let w', be any world accessible from w, i.e., wRw'
5. from the fact that R is symmetric, we have that $w'Rw$
6. From the satisfiability condition of $\Diamond$, from the fact that $w'Rw$ and that $M, w \vDash \varphi$, we have that $M, w' \vDash \Diamond\varphi$.
7. so for every world w' accessible from w, we have that $M, w' \vDash \Diamond\varphi$.
8. From the satisfiability condition of $\Box$, $M, w \vDash \Box\Diamond\varphi$ (Thesis)
9. Since from (Hypothesis) we have derived (Thesis), we can conclude that $M, w \vDash \varphi \supset \Box\Diamond\varphi$.

R is symmetric - completeness

Suppose that a frame $F = (W, R)$ is not Symmetric.

1. If R is not symmetric then there are two worlds $w, w' \in W$ such that wRw' and not $w'Rw$
2. Let M be any model on F, and let φ be the propositional formula p. Let V the set p false in all the worlds of W but w where p is set to be true.
3. From the fact that w' does not access to w , it means that in all the worlds accessible from w', p is false,
4. i.e. there is no world w'' accessible from w' wuch that $M, w'' \vDash p$.
5. by the satisfiability conditions of $\Diamond$, we have that $M, w' \nvDash \Diamond p$.
6. Since there is a world w' accessible from w , with $M, w \nvDash \Diamond p$, form the satisfiability condition of $\Box$ we have that $M, w \nvDash \Box\Diamond p$.
7. since $M, w \vDash p$, and $M, w \nvDash \Box\Diamond p$. we have that $M, w \nvDash p \supset \Box\Diamond p$.

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