Mathematical Logics
Modal Logic: K and more*

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Properties of accessibility relation

Formulas can be used to shape the "form" of the structure, namely to impose properties on the accessibility relation R .

Examples

- ***Temporal logic***: if the accessibility relation is supposed to represent a temporal relation, and wRw^l means that w^l is a future world w.r.t. w, then R must be a transitive relation. That is if w^l is a future world of w, then any future world of w^l is also a future world of w .

- ***Logic of knowledge***: if the accessibility relation is used to represent the knowledge of an agent A, and wRw^l represents the fact that w^l is a possible situation coherent with its actual situation w , then R must be reflexive, since w is always coherent with itself.

Typical Properties of R

The following table summarizes the most relevant properties of the accessibility relation, which have been studied in modal logic, and for which it has been provided a sound and complete axiomatization

Properties of R

R is reflexive	$\forall w.R(w, w)$
R is transitive	$\forall w\ v\ u.(R(w, v) \wedge R(v, u) \supset R(w, u))$
R is symmetric	$\forall w\ v.(R(w, v) \supset R(v, w))$
R is Euclidean	$\forall w\ v\ u.(R(w, v) \wedge R(w, u) \supset R(v, u))$
R is serial	$\forall w.\exists v R(w, v)$
R is weakly dense	$\forall w\ v.R(w, v) \supset \exists u.(R(w, u) \wedge R(u, v))$
R is partly functional	$\forall w\ v\ u.(R(w, v) \wedge R(v, u) \supset v = u)$
R is functional	$\forall w\ \exists! v.R(w, v)$
R is weakly connected	$\forall u\ v\ w.(R(u, v) \wedge R(u, w) \supset R(v, w) \vee v = w \vee R(w, v))$
R is weakly directed	$\forall u\ v\ w.(R(u, v) \wedge R(u, w) \supset \exists t(R(v, t) \wedge R(w, t)))$

We will investigate only the ones in red.

Soundness and completeness

Modal logics vs. properties of accessibility relations

K		the class of all frames
K4	**4**	the class of transitive frames
KT	**T**	the class of reflexive frames
KB	**B**	the class of symmetric frames
KD		the class of serial frames
KT4	**S4**	the class of reflexive and transitive frames
KT4B	**S5**	the class of frames with an equivalence relation
KT5	**S5**	the class of frames with an equivalence relation

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