Mathematical Logics
Modal Logic: K and more*

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Lecture index

- **Hilbert calculi:** used for properties of logics

- **Tableau calculus:** used for reasoning

- *This Lecture:* the family of propositional modal logics

- *Next Lecture:* Tableaux calculus

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A1	$\varphi \supset (\psi \supset \varphi)$
A2	$(\varphi \supset (\psi \supset \vartheta)) \supset ((\varphi \supset \psi) \supset (\varphi \supset \vartheta))$
A3	$(\neg\psi \supset \neg\varphi) \supset ((\neg\psi \supset \varphi) \supset \varphi)$
MP	$\dfrac{\varphi \quad \varphi \supset \psi}{\psi}$
K	$\Box(\varphi \supset \psi) \supset (\Box\varphi \supset \Box\psi)$
Nec	$\dfrac{\varphi}{\Box\varphi}$ the necessitation rule

Necessitation rule can be applied only if premise if a theorem

The above set of axioms and rules is called **K**, and every modal logic with a validity relation closed under the rules of **K** is a Normal Modal Logic.

Notice that **Nec** rule is not the same as

$$\varphi \supset \Box\varphi$$

(3)

indeed formula (3) is not valid.

Assignment Find a model in which (3) is false. Hint: build model with
a model where φ is true and $\Box\varphi$ is false   (namely $\neg\Box\varphi$ is true)

Nec: In Hilbert calculi there is difference from deduction and proof.
Deduction theorem does not hold

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