Mathematical Logics

FOL: Reasoning as deduction

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Lecture index

1. Reasoning problems (recap)
2. Hilbert systems (VAL – forward chaining)
3. Tableaux systems ((un)-SAT – backward chaining)
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Saturated Branches

Saturated open

An open branch is called saturated if every non-literal has been analyzed at least once and, additionally, every γ-formula (γ-formulas are of the form $\forall x\varphi$ and $\neg\exists x\varphi$) has been instantiated with every term we can construct using the function symbols on the branch.

Failing proof

A tableau with an open saturated branch can never be closed, i.e. we can stop an declare the proof a failure.

Is this the solution?

This only helps us in special cases though. (A single 1-place function symbol together with a constant is already enough to construct infinitely many terms . . .)

Countermodels

- If the construction of a tableaux ends in a saturated open branch, the tableaux can be used to define the interpretation which is also a model *M* for all the formulas on that branch.
- *M is finite* by construction. It is a subset of other *possibly infinite* models.
- A model *M, being an interpretation, must tell how to interpret constants* (the elements of the *domain*), *function symbols, and predicate symbols*
- *Domain*: set of all terms we can construct using the function symbols appearing on the branch (so-called *Herbrand universe*). (You can optionally introduce a fake constant for the value of the term)
- *Function symbols*: interpreted as themselves (or using the fake constants)
- *Predicate symbols*: interpreted in terms of their occurrences in the branch

Example

Example

$\exists x\,(P(x) \wedge \neg Q(x)) \wedge \forall y\,(P(y) \vee Q(y))$

$\exists x\,(P(x) \wedge \neg Q(x))$
$\forall y\,(P(y) \vee Q(y))$

$P(a) \wedge \neg Q(a)$

$P(a)$
$\neg Q(a)$

$P(a) \vee Q(a)$

$P(a)$ | $Q(a)$

OPEN | CLASH

Comments

From the formulas appearing in the OPEN branch of the tableaux it is possible to construct a model for the root formula.

- $\Delta = \{a\}$, the constants appearing in the formulas
- $I\,(P) = \{a\}$, since the formula $P(a)$ appears in the open branch
- $I\,(Q) = \{\}$ since the formula $\neg Q(a)$ appears in the open branch

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