Mathematical Logics
FOL: Reasoning as deduction

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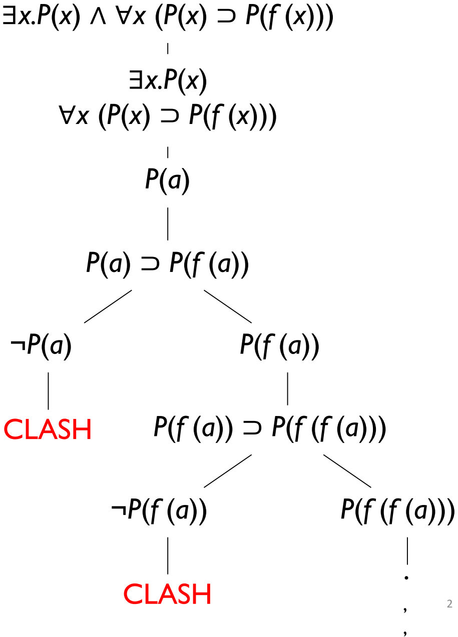
Lecture index

1. Reasoning problems (recap)
2. Hilbert systems (VAL – forward chaining)
3. Tableaux systems ((un)-SAT – backward chaining)
4. Correctness and completeness of Tableau
5. Examples
6. Termination
7. Countermodels

Problem of (non) Termination

For certain formulas there is the possibility of infinite branches

Key role of function symbols as generators of an unbound number of terms



Infinite domains

Differently from Prop. Logic, in FOL, models can be infinite.

There are formulas which are satisfied only by infinite models. For instance the following formula[1]

$$\varphi = \begin{pmatrix} \forall x\, \neg R(x, x) & \wedge \\ \forall xyz.(R(x, y) \wedge R(y, z) \supset R(x, z)) & \wedge \\ \forall x.\exists y.R(x, y) & \end{pmatrix}$$

Existential quantifier behaves like a function symbol

If we build a tableaux for such a formula, searching for a model, we will end up in an infinite tableaux.

Infinite tableaux

Exercize

Build a tableaux for
$\forall x\, \neg R(x, x) \wedge \forall xyz.(R(x, y) \wedge R(y, z) \supset R(x, z)) \wedge \forall x.\exists y.R(x, y)$

Solution

$$\forall x\, \neg R(x, x) \wedge \forall xyz.(R(x, y) \wedge R(y, z) \supset R(x, z)) \wedge \forall x.\exists y.R(x, y)$$

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$$\forall x\, \neg R(x, x)$$
$$\forall xyz.(R(x, y) \wedge R(y, z) \supset R(x, z))$$
$$\forall x.\exists y.R(x, y)$$

|

$$\exists y.R(a_0, y)$$

|

$$R(a_0, a_1)$$

|

$$\exists y.R(a_1, y)$$

|

⋮

By applying the γ-rule to the axiom $\forall x \exists y (R(x, y))$, *we generate* $\exists y R(a_0, y)$ *for an initial constant* a_0, *and by applying the δ-rule to this last formula we generate a new individual* a_1 *This allow to apply the γ-rule again to* $\forall x\, \exists y R(x, y)$, *obtaining* $\exists y R(a_1, y)$, *and again by applying δ-rule to this new formula we generate another constant* a_2.
The process can go on infinitively without reaching any clash

Termination of a FOL tableaux

- In contrast to what happens in propositional logic, the tableaux construction is not guaranteed to terminate.
- ***If the formula φ*** that labels the root is ***unsatisfiable***, in which case the construction is guaranteed to terminate and the tableau can be closed.
- ***If the formula φ*** that labels the root is ***satisfiable*** then either the construction is guaranteed to terminate and the tableau is open, or the construction does not terminate.
- ***SEARCH PROBLEM:*** if you have not yet been able to close the tableaux, is it because the formula is satisfiable or because you have not found the way to construct the tableaux? ***You cannot know!*** (the search dilemma)

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