Mathematical Logics

FOL: Reasoning as deduction

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1. Reasoning problems (recap)
2. Hilbert systems (VAL – forward chaining)
3. Tableaux systems ((un)-SAT – backward chaining)
4. Correctness and completeness of Tableau
5. Examples
6. Termination
7. Countermodels

Reasoning problems in FOL

In FOL we have three main reasoning problems (as in all logics)

Satisfiability

Find an interpretation *I* that satisfies a closed formula φ. I.e., check if there is a I such that I $\models \varphi$.

Validity

Check if a formula φ is valid, i.e., if for all interpretations *I*, $I \models \varphi$

Logical consequence

Check if a formula φ is a logical consequence of a set of formulas Γ, i.e.,

$\Gamma \models \varphi$

- Model checking done mainly in finite domains where it reduces to PL Model checking (see before). For model checking in infinite domains take Formal Methods course (main applications: SW and HW verification)
- Logical Equivalence via Logical Consequence

FOL Properties (Recap)

Proposition

A *Valid* $\longrightarrow A$ *satisfiable* $\longleftrightarrow A$ *not unsatisfiable*

A *unsatisfiable* $\longleftrightarrow A$ *not satisfiable* $\longrightarrow A$ *not Valid*

$\Gamma, A \models B \longleftrightarrow \Gamma \models A \rightarrow B$

$\Gamma \models \varphi \longleftrightarrow \Gamma \cup \{\neg\varphi\}$ *not satisfiable*

Proposition

if A is	*then ¬A is*
Valid	*Unsatisfiable*
Satisfiable	*not Valid*
not Valid	*Satisfiable*
Unsatisfiable	*Valid*

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Hilbert style axiomatization

Axioms for propositional connectives (the same as in PL)

A1 $\varphi \supset (\psi \supset \varphi)$

A2 $(\varphi \supset (\psi \supset \vartheta)) \supset ((\varphi \supset \psi) \supset (\varphi \supset \vartheta))$

A3 $(\neg\psi \supset \neg\varphi) \supset ((\neg\psi \supset \varphi) \supset \psi)$

MP $$\frac{\varphi \quad \varphi \supset \psi}{\psi}$$

Axioms and rules for quantifiers

A4 $\forall x.(\varphi(x)) \supset \varphi(t)$ if t is free for x in $\varphi(x)$

A5 $\forall x.(\varphi \supset \psi) \supset (\varphi \supset \forall x.\psi)$ if x does not occur free in φ

Gen $$\frac{\varphi}{\forall x.\varphi}$$

NOTE: Hilbert FOL is correct and complete with respect First order semantics

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