Mathematical Logics
First Order Logic*

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1 Lecture index

1. Intuition
2. Language
3. Interpretation function
4. Satisfiability with respect to an assignment
5. Satisfiability, Validity, Unsatisfiability, Logical Conseguence and Logical Equivalence
6. Exercizes
7. Finite domains
8. Analogy with data bases

Finite domain, with names for every element

Unique Name Assumption (UNA)

Is the assumption under which the language contains a name for each element of the domain, i.e., the language contains the constant $c_1, \ldots, c_n$, and each constant is the name of one and only one domain element.

$$\varphi_{\Delta=\{c_1,\ldots,c_n\}} = \left(\bigwedge_{i \neq j=1}^{n} c_i \neq c_j \wedge \forall x \left(\bigvee_{i=1}^{n} c_i = x \right) \right)$$

$\varphi_{\Delta=\{c_1,\ldots,c_n\}}$ *is called the Unique Name Assumption.*

NOTE: constants are also elements of the domains

Finite predicate extension

The assumption that states that a predicate P is true only for a finite set of objects for which the language contains a name, can be formalized by the following formulas:

$$\forall x.(P(x) \equiv (x = c_1 \vee \ldots \vee x = c_n))$$

Example

- The days of the week are: Monday, Tuesday, . . . , Sunday.

  $$\forall x.(\text{WeekDay}(x) \equiv x = \text{Mon} \vee x = \text{Tue} \vee \ldots \vee x = \text{Sun})$$

- The WorkingDays are: Monday, Tuesday, . . . , Friday:

  $$\forall x.(\text{WorkingDay}(x) \equiv x = \text{Mon} \vee x = \text{Tue} \vee \ldots \vee x = \text{Fri})$$

Finite domain - Grounding

Under the hypothesis of finite domain with a constant name for every elements, First order formulas can be be propositionalized, aka grounded as follows:

$$\varphi_{\Delta=\{c_1,\dots,c_n\}} \models \forall x.\varphi(x) \equiv \varphi(c_1) \wedge \dots \wedge \varphi(c_n) \quad (1)$$

$$\varphi_{\Delta=\{c_1,\dots,c_n\}} \models \exists x.\varphi(x) \equiv \varphi(c_1) \vee \dots \vee \varphi(c_n) \quad (2)$$

Generalizing:

$$\varphi_{\Delta=\{c_1,\dots,c_n\}} \models \forall x_1 \dots x_k . \varphi(x_1,\dots,x_k) \equiv \bigwedge_{\substack{c_{i_1},\dots,c_{i_k} \in \\ \{c_1,\dots,c_n\}}} \varphi(c_{i_1},\dots,c_{i_k}) \quad (3)$$

$$\varphi_{\Delta=\{c_1,\dots,c_n\}} \models \exists x_1 \dots x_k . \varphi(x_1,\dots,x_k) \equiv \bigvee_{\substack{c_{i_1},\dots,c_{i_k} \in \\ \{c_1,\dots,c_n\}}} \varphi(c_{i_1},\dots,c_{i_k}) \quad (4)$$

NOTE: grounding allows to reduce FOL Reasoning to PL reasoning, e.g., Use of DPLL..
Try exercizes with simple formulas and truth tables.

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