Mathematical Logics
First Order Logic*

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Modified by Fausto Giunchiglia and Mattia Fumagalli

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FOL Interpretation function

FOL interpretation for a language L

A first order interpretation for the language $L = (c_1, c_2, \ldots, f_1, f_2, \ldots, P_1, P_2, \ldots)$ is a pair (Δ, I) where

- Δ is a non empty set called interpretation domain
- I is is a function, called interpretation function
  - $I(c_i) \in \Delta$ (elements of the domain)
  - $I(f_i) : \Delta^n \rightarrow \Delta$ (n-ary function on the domain)
  - $I(P_i) \subseteq \Delta^n$ (n-ary relation on the domain)

  where n is the arity of f_i and P_i.

Interpretation function I: L → D

Example of interpretation

Example (Of interpretation)

Symbols

Constants: *alice*, *bob*, *carol*, *robert*
Function: *mother-of* (with arity equal to 1)
Predicate: *friends* (with arity equal to 2)

Domain $\Delta = \{1, 2, 3, 4, \ldots\}$

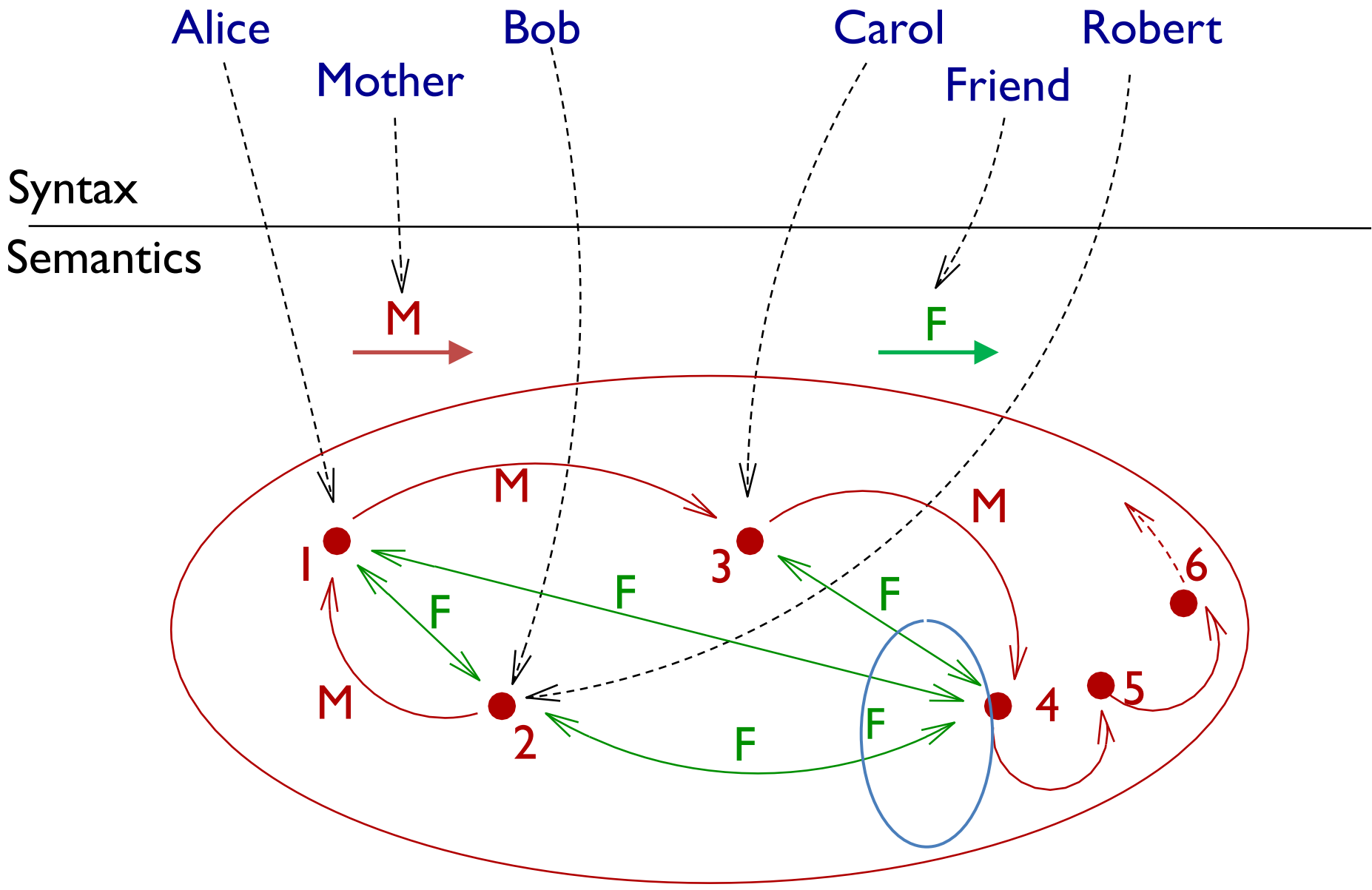
Interpretation $I(\textit{alice}) = 1$, $I(\textit{bob}) = 2$, $I(\textit{carol}) = 3$, $I(\textit{robert}) = 2$

$I(\textit{mother-of}) = M$

$$M(1) = 3$$
$$M(2) = 1$$
$$M(3) = 4$$
$$M(n) = n + 1 \text{ for } n \geq 4$$

$$I(\textit{friends}) = F = \left\{ \begin{array}{lll} \langle 1,2\rangle, & \langle 2,1\rangle, & \langle 3,4\rangle, \\ \langle 4,3\rangle, & \langle 4,2\rangle, & \langle 2,4\rangle, \\ \langle 4,1\rangle, & \langle 1,4\rangle, & \langle 4,4\rangle \end{array} \right\}$$

Example (cont'd)



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