

Mathematical Logics

First Order Logic*

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Modified by Fausto Giunchiglia and Mattia Fumagalli*

1 Lecture index

1. Intuition
2. Language
3. Interpretation function
4. Satisfiability with respect to an assignment
5. Satisfiability, Validity, Unsatisfiability, Logical Consequence and Logical Equivalence
6. Exercises
7. Finite domains
8. Analogy with data bases

FOL interpretation for a language L

A first order interpretation for the language

$L = (c_1, c_2, \dots, f_1, f_2, \dots, P_1, P_2, \dots)$ is a pair (Δ, I) where

- Δ is a non empty set called **interpretation domain**
- I is a function, called **interpretation function**
 - $I(c_i) \in \Delta$ (elements of the domain)
 - $I(f_i) : \Delta^n \rightarrow \Delta$ (n -ary function on the domain)
 - $I(P_i) \subseteq \Delta^n$ (n -ary relation on the domain)

where n is the arity of f_i and P_i .

Interpretation function $I: L \rightarrow D$

Example of interpretation

Example (Of interpretation)

Symbols

Constants: *alice*, *bob*, *carol*, *robert*

Function: *mother-of* (with arity equal to 1)

Predicate: *friends* (with arity equal to 2)

Domain

$$\Delta = \{1, 2, 3, 4, \dots\}$$

Interpretation

$$I(\textit{alice}) = 1, I(\textit{bob}) = 2, I(\textit{carol}) = 3, \\ I(\textit{robert}) = 2$$

$$I(\textit{mother-of}) = M$$

$$M(1) = 3$$

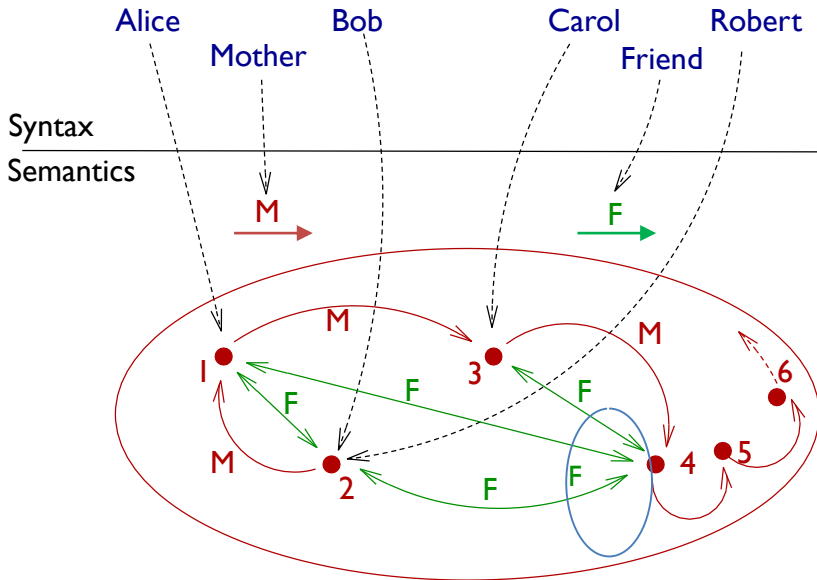
$$M(2) = 1$$

$$M(3) = 4$$

$$M(n) = n + 1 \text{ for } n \geq 4$$

$$I(\textit{friends}) = F = \left[\begin{array}{ccc} \langle 1, 2 \rangle, & \langle 2, 1 \rangle, & \langle 3, 4 \rangle, \\ \langle 4, 3 \rangle, & \langle 4, 2 \rangle, & \langle 2, 4 \rangle, \\ \langle 4, 1 \rangle, & \langle 1, 4 \rangle, & \langle 4, 4 \rangle \end{array} \right]$$

Example (cont'd)



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